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Predicting and decomposing why representative agent and heterogeneous agent models sometimes diverge☆

Trevor S. Gallen

Purdue University Krannert School of Business, West Lafayette, IN, 47906, USA

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ABSTRACT

Heterogeneous agent (HA) and representative agent (RA) models often give very different answers to important economic questions, even when studying the same phenomenon with the otherwise identical assumptions. There are a variety of reasons why HA and RA models may differ. This paper provides a formula that can be used to decompose the different response functions of HA and RA models into six major “mechanical” categories. Because solving HA models is costly, researchers using this formula are better able to predict when an HA model is unlikely to improve upon an RA model, and defend their use of an RA model without first solving the HA model. Moreover, when two models do diverge, this paper provides a framework for understanding mechanically why they diverged.

1. Introduction

Understanding the differences and similarities between heterogeneous agent (HA) models and representative agent (RA) models has been at the forefront of macroeconomics for the last twenty years. While it is known that perfect macroeconomic aggregation can fail for even trivial reasons, this does not preclude use of RA abstraction if the divergence implied for aggregate behavior is small. This is perhaps most famously the case for Krusell and Smith (1998)’s “approximate aggregation” in a business-cycle environment with heterogeneous agents and incomplete capital markets. However other models (such as Heathcote (2005), and Chang and Kim (2007)), find failures of approximate aggregation in comparable model by adding a labor supply choice and incomplete markets. Understanding precisely why two models give different responses is typically treated as model-idiosyncratic.

The purpose of this paper is to enable researchers to cut through model-idiosyncratic discussions of potential HA and RA differences and to provide a common framework for understanding potential divergences. To do so, I take an exogenous variable of interest (such as tax rates), and examine the representative agent’s optimal policy for an endogenous variable of interest (such as labor hours) as a function of that variable. Next, I quadraticize the endogenous response function. I decompose the response into six distinct, easily-calculable elements. These elements measure the heterogeneity in linear and quadratic responses of agents to treatment interacted with heterogeneity in treatment. Isolating these six individually allows a researcher to clearly analyze where heterogeneity in treatment potentially interacts with heterogeneity in linear and quadratic responsiveness in economic agents.

The six reasons for divergence each have clear intuition. The first two are obvious, but important. When agents who are more responsive to a given treatment variable also experience larger changes in that variable, a representative agent model will misstate overall response. As a concrete example, I find this is particularly important in a heterogeneous agent lifecycle model, in which older, wealthier agents are both more elastic and receive a larger share of a tax cut calibrated to the Tax Cuts and Jobs Act of 2017 (hereafter TCJA 2017). Third, even if agents are homogeneous in their quadratic response to a variable, heterogeneity in that variable will cause divergence. This is the case, for instance, with binary labor choice models as in Chang and Kim (2007). Fourth, and perhaps most subtly, when the third mixed moment of quadratic responsiveness and treatment is nonzero, divergences may

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E-mail address: tgallen@purdue.edu.

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occur. This occurs when there is variance in the quadratic responsiveness that covaries with the square of a variable change, such as in the binary labor choice model. Linear and quadratic initial miscalibration of a representative agent model provide the fifth and sixth potential reasons. While this miscalibration or responsiveness may seem trivial, it occurs frequently when calibrating to the level of a variable, as is done in macroeconomics. Specifically, response elasticities frequently change as functions of parameters used to calibrate to levels in an HA environment. This is the case for instance with standard log preferences when agents have been perturbed off a balanced growth path, as heterogeneous agents in stochastic models almost always are.

This paper makes two important contributions. First, it offers a method to predict when RA models will diverge from HA models without first calculating the HA model. Second, it allows for a quantitative decomposition of the source of expected divergence. To illustrate the usefulness of this method, this paper takes the formula that describes the sources of divergence and applies it to a modern macroeconomic dynamic general equilibrium model. Specifically, it takes the lifecycle model of human capital accumulation used in Badel and Huggett (2017) and applies it to a stylized version of the Tax Cuts and Jobs Act of 2017 (hereafter TCJA). In discussing preferences, it shows that a researcher can predict strong divergence from a representative agent model based simply on sensitivity analysis of preferences.

The method I propose in this paper is useful primarily in predicting and diagnosing very short- and very long-run divergences between models because it quadraticizes agent response functions response to a single shock around a steady state. It is of more limited applicability in analyzing the dynamics of complicated macroeconomic models because the interactions of differentially evolving state variables are difficult to capture in a simple formula. For instance, while I use it to understand the immediate and long-run effects of a tax change, it is not well-suited to decompose divergence between RA and HA models along the transition path.

This paper's methodology complements a recent vein of research that studies the difference between RA and HA models. An et al. (2009) study the economy of Chang and Kim (2007) and argue a HA model's first optimization conditions do not aggregate well because heterogeneous agents are often at corner solutions in terms of labor choice. Heathcote (2005) shows that the propensity to consume out of a temporary tax cut with lump-sum taxes is 13.5% in a heterogeneous agent model with incomplete markets, while it is zero in a representative agent/complete markets model. Buera and Moll (2015) show a representative agent model is limited in its ability to capture the effects of a credit crunch over the business cycle. Kaplan et al. (2018) find that heterogeneous marginal propensities to consume are crucial in understanding the effects of interest rate cuts on consumption behavior. Ahn et al. (2018) finds that realistic modeling of consumption response to predictable aggregate income changes similarly requires heterogeneity. Misra and Surico (2014) estimate a significant degree of heterogeneity in responsiveness to tax rebates and reject a homogenous response model. Others, such as Nakajima (2005), Braun and Nakajima (2012), and Werning (2015) find that under certain conditions, a representative agent model with an adjusted discount rate might well mimic consumption dynamics in a model with incomplete markets. This paper differs from prior research by taking a mechanical, rather than economic route to understanding differences between RA and HA models.

Most analyses of divergence between RA and HA models study them by changing various components of a model, such as discrete labor supply, borrowing constraints, incomplete markets, or the calibration of parameters. This paper studies the mechanical, rather than economic reasons for divergence, and gives a quantitative way to analyze the degree to which miscalibration, heterogeneous linearities in response, nonlinearities in response, and heterogeneity in exposure to a shock or treatment causes differences between two models. This method is particularly useful when there are multiple sources contributing to a divergence, and allows for more broad generalizations and categorization of the sources of divergence. The benefit of this paper's approach is that it allows insight into when heterogeneous agent models might diverge from representative agent models before calculating the HA model.

The rest of this paper proceeds as follows. Section 2 derives the set of formulae that describe the sources of divergence. Section 3 uses the insights from Section 2 to predict when using a standard representative agent model to predict the effects of a tax cut will diverge from a heterogeneous agent model. Section 4 sets out a modern dynamic lifecycle model of work, consumption, and human capital formation based on Badel and Huggett (2017), uses it to predict the effects of the TCJA 2017 on earnings, and applies this paper's methodology to that model. Section 5 concludes.

2. Model

Assume a single univariate outcome of interest, which for convenience is the change in aggregate labor hours (ΔL), but could be any endogenous outcome. The 'stimulus" or "shock" or "treatment" being analyzed is a marginal tax rate change ($\Delta\tau$). A researcher is interested in $\Delta L(\Delta\tau)$, the change in labor generated by a change in marginal tax rate. While simple, this setup encompasses the wide range of models and analyses that focus on a small subset of outcomes, such as the response of aggregate labor hours or consumption to a productivity shock or a tax/transfer change. It similarly encompasses studies that analyze the response of a set of macroeconomic aggregates to a single exogeneous shock without decomposing endogenous dependencies, as in the case of impulse response functions in macroeconomic analyses. This setup allows for both types of analyses. For a heterogeneous agent model with N agents, I write the aggregate ΔL^H as the sum of individual responses to a change in own taxes τ_i:

$$\Delta L^H = \sum_{i=1}^{N} \Delta L_i^H(\Delta\tau_i)$$

Note that even if agents have identical preferences and marginal tax change $\Delta\tau_i$, change in labor $\Delta L_i(\Delta\tau_i)$ may differ across agents because of asset differences in the face of borrowing constraints, wage differences, non-labor income differences, or initial tax differences.

The analogous representative agent is simply denoted as $\Delta L^R(\Delta\tau)$. Let $\beta^j = \frac{\partial L^j}{\partial\tau}$ and $\gamma^j = \frac{1}{2}\frac{\partial^2 L^j}{\partial\tau^2}, j \in \{R, H\}$ for the relevant RA or HA model respectively. Define second-order approximations (denoted with hats) of both the representative agent and heterogeneous agent responses to a non-infinitesimal change in taxation $\Delta\tau$.

$$\widehat{\Delta L^R} = \beta^R \Delta\tau^R + \gamma^R \left(\Delta\tau^R\right)^2 \tag{1}$$

$$\widehat{\Delta L_i^H} = \beta_i^H \Delta\tau_i^H + \gamma_i^H \left(\Delta\tau_i^H\right)^2 \quad i \in \{1, \dots N\} \tag{2}$$

where the first and second terms on the RHS of each equation distinguish linear and quadratic responsiveness to marginal tax rate changes, while R and H denote the representative agent and heterogeneous agent models. Note that in this formulation, I assume that an agent's responsiveness is dependent only on their own tax rate change, not the distribution of changes.[1] I discuss how endogenous changes may be incorporated by adding another term to (2) later in this section. The core contribution of this paper is to offer a closed form, interpretable expression to quantify how Equation (1) and the sum of many agents responses described by Equation (2) diverge. Fig. 1 depicts the four models (two full models, and two quadratic approximations) and the

[1] It is possible to choose β_i^H and γ_i^H specific to a policy experiment, describing the full effect of a policy, rather than the partial effect, so that general equilibrium responsiveness is loaded onto these coefficients. For instance, take a case in which an agent is highly responsive to $\Delta\tau_i$ ceteris paribus, but their response to taxes is entirely offset by endogenous changes in the wage in such a case, a good approximation to the experiment would set $\beta_i = \gamma_i = 0$.

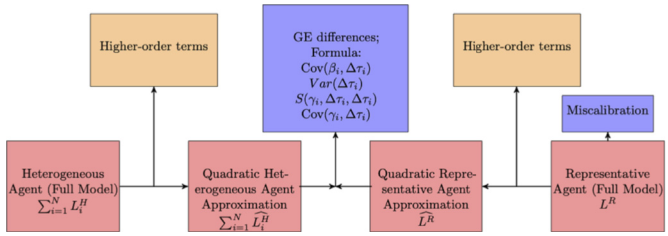

Fig. 1. This bottom half of this figure depicts the four possible models: a representative agent model, a heterogeneous agent model, and their two quadratic approximations. The top half describes the causes of divergence between representative agent and heterogeneous agent models discussed in this paper.

potential sources of divergence when translating from model to model that this section derives.

As I discuss, the representative agent model may be wrong initially due to miscalibration or because endogenous equilibrium objects differ. Miscalibration can happen, for instance, when a functional form is not flexible enough to properly allow calibration to responsiveness. I argue this is potentially quite common in calibrated macroeconomic models, because β^R and γ^R are functions of parameters that are calibrated to target the levels of outcomes, but affect elasticities of responses. In such a case, naive calibration to the levels of an economy's aggregate variables in a representative budget constraint will cause significant initial errors. I discuss this possibility in the next section.

Different endogenous responses between models can happen when endogenous prices, such as wages, respond more strongly in one model than another, such as when wages decline following a shift out in labor supply, partially muting the response of labor quantity supplied. Even when properly calibrated with no general equilibrium differences, there is some loss of information when translating either the representative agent or the heterogeneous agent model to their quadraticized counterparts due to higher-order terms. Finally, there are four possible sources of error when comparing the two quadraticized agents which I derive below.

Assuming that a heterogeneous agent model is correct, a representative agent model's quadratic approximation could approximate the heterogeneous agent model's quadratic approximation by taking the same model of individuals and applying it to one "representative" individual,[2] so that Equation (1) becomes:

$$\widehat{\Delta L^R} = \overline{\beta} \cdot \overline{\Delta\tau} + \overline{\gamma}\left(\overline{\Delta\tau}\right)^2 \tag{3}$$

There are many potential choices for $\overline{\beta}$, $\overline{\gamma}$, and $\overline{\Delta\tau}$. First examine the obvious choices, in which each is replaced by its sample mean, which I show may yield significant deviations between HA and RA models:

$$\overline{\beta} = \frac{1}{N}\sum_{i=1}^{N}\beta_i \qquad \overline{\gamma} = \frac{1}{N}\sum_{i=1}^{N}\gamma_i \qquad \overline{\Delta\tau} = \frac{1}{N}\sum_{i=1}^{N}\Delta\tau_i \tag{4}$$

In such a case, the difference between the average heterogeneous agent response, derived by the mean of all agents in Equation (2) and the representative agent's response, summarized by Equation (1), can be solved for in closed form.

To do so, I note that there are several convenient ways to rewrite the sample means $(\Delta\tau_i)^2$, $\beta_i\Delta\tau_i$, and $\gamma_i\Delta\tau_i$ present in Equation (2) using the sample means of $\Delta\tau$, β, γ, as well as the variances, covariances, and co-skewness of the terms.[3] Specifically, it can be shown that:

$$\frac{1}{N}\sum_{i=1}^{N}\beta_i\Delta\tau_i = \overline{\beta}\cdot\overline{\Delta\tau} + Cov(\beta_i, \Delta\tau_i)$$

and

$$\frac{1}{N}\sum_{i=1}^{N}\gamma_i\Delta\tau_i^2 = \overline{\gamma}\,(\overline{\Delta\tau})^2 + 2\overline{\Delta\tau}Cov(\gamma_i, \Delta\tau_i) + \overline{\gamma}Var(\Delta\tau_i) + Var(\Delta\tau_i)\sqrt{Var(\gamma_i)}S(\gamma_i, \Delta\tau_i, \Delta\tau_i)$$

Noting that the first term in each of the above approximations is a term in the calibrated representative agent's response, write the difference in heterogeneous response and the representative agent's response as:

$$\widehat{\Delta L^H} - \widehat{\Delta L^R} = Cov(\beta_i, \Delta\tau_i) + \overline{\gamma}Var(\Delta\tau_i) + Var(\Delta\tau_i)\sqrt{Var(\Delta\gamma_i)}S(\gamma_i, \Delta\tau_i, \Delta\tau_i) + 2\overline{\Delta\tau}Cov(\gamma_i, \Delta\tau_i) \tag{5}$$

Where ΔL^R is the naively-calibrated representative agent of Equations (3) and (4). Equation (5) describes the difference in responsiveness between a heterogeneous agent and representative agent economy with the "proper" calibrations given by Equation (4) and no difference in the exogenous variable $\Delta\tau_i$. I now turn to two important extensions.

The formula in Equation (5) can be further extended to capture two additional important forces which may cause HA and RA models to diverge. First, it is possible that a researcher does not calibrate the two models according to the specification in Equation (4). Indeed, as I discuss in the next section, this happens either when researchers calibrate parameters that affect responsiveness to levels, or do not have enough functional form flexibility to match all targets. In such a case, two new "miscalibration" terms may be added, summarizing the difference between the "properly calibrated" model assumed by Equation (4) and the one used by a researcher. Let $\overline{\beta}^{MC}$ and $\overline{\gamma}^{MC}$ be the values for β and γ that deviate from those given in Equation (4). Correcting for miscalibration is straightforward, and is given in Equation (6).

$$\widehat{\Delta L^R} - \widehat{\Delta L^{MC}} = (\overline{\beta} - \overline{\beta}^{MC})\Delta\tau + (\overline{\gamma} - \overline{\gamma}^{MC})(\overline{\Delta\tau})^2 \tag{6}$$

Moreover, there may be a general equilibrium difference in another variable of interest between two models. For instance, we might be concerned that a tax reform affects the level of wages (common throughout the economy) and affects the quantity of labor supplied. By assuming a constant miscalibration of $\Delta\tau$ or a constant change in an endogenous variable such as wages, four new terms summarize the difference between the full representative response ΔL^R and the "simply calibrated" response $\Delta\overline{L}^{SC}$. Letting T be the uniform misstatement of the

[2] The representative agent's squared term squares the same term that $\overline{\beta}$ multiplies: an ordinary taylor approximation might instead choose some more representative $\overline{(\Delta\tau)^2} \neq \left(\overline{\Delta\tau}\right)^2$ for the squared term. In most models, however, there is only one $\Delta\tau$ input parameter, and the curvature is dictated by the model's quadratic reaction to $\Delta\tau$, rather than having two separate parameters. For instance, the representative budget constraint in a static environment: $wL(1-\tau)+v = c$allows for only one τ parameter, with no explicitly separate linear and quadratic tax entry.

[3] Co-skewness is the third mixed moment, defined as: $S(\gamma_i, \Delta\tau_i, \Delta\tau_i) = \frac{1}{N}\sum_{i=1}^{N}\frac{((\Delta\tau_i - \overline{\Delta\tau})^2(\gamma_i - \overline{\gamma}))}{Var(\Delta\tau_i)\sqrt{Var(\gamma_i)}}$

Table 1
Summary of representative agent misstatement of aggregate response.

	Concept	Relevant Terms	Misstatement
(1)	Heterogeneous taxes, homogenous nonlinear response	$\Delta\tau_i$, $\overline{\gamma} \neq 0$	$\overline{\gamma} Var(\Delta\tau_i)$
(2)	Heterogeneous taxes, heterogeneous linear response	$\Delta\tau_i$, β_i	$Var(\Delta\tau_i)\sqrt{Var(\gamma_i)}S(\gamma_i, \Delta\tau_i, \Delta\tau_i)$ $+Cov(\beta_i, \tau_i)$
(3)	Heterogeneous taxes, heterogeneous nonlinear response	$\Delta\tau_i$, γ_i	$2\overline{\tau}Cov(\gamma_i, \tau_i)$
(4)	Endogenous (homogeneous) differences in $\Delta\tau$	$T \neq 0$	$\overline{\beta}T + \overline{\gamma}T^2$ $+\gamma\overline{\tau}T + T \cdot Cov(\gamma_i, \tau_i)$
(5)	Linear responsiveness misstatement	$\overline{\beta} \neq \frac{1}{N}\sum_{i=1}^{N}\beta_i$	$\left(\frac{1}{N}\sum_{i=1}^{N}\beta_i - \overline{\beta}\right)\overline{\Delta\tau}$
(6)	Quadratic responsiveness misstatement	$\overline{\gamma} \neq \frac{1}{N}\sum_{i=1}^{N}\gamma_i$	$\left(\frac{1}{N}\sum_{i=1}^{N}\gamma_i - \overline{\gamma}\right)\overline{\Delta\tau}^2$

Table 1: This table summarizes how a representative agent's univariate response can deviate from a heterogeneous agent's univariate response, summarizing Equations (5)–(7). β denotes the linear responsiveness with respect to $\Delta\tau$, γ denotes quadratic responsiveness, T denotes the homogeneous misstatement of $\Delta\tau$. In the notation above, $\overline{\beta}$ and $\overline{\gamma}$ refer to the "correctly" calibrated except in lines (5) and (6).

endogenous variable of interest, Equation (7) describes the new terms involved in divergence.

$$\widehat{\Delta L^R} - \widehat{\Delta L^{SC}} = \overline{\beta}T + \overline{\gamma}T^2 + 2T \cdot \overline{\gamma} \cdot \overline{\tau} + 2TCov(\gamma_i, \tau_i) \tag{7}$$

Table 1 collects the results of Equations (5)–(7), and describes the misstatement terms relevant in a representative agent model, depending on whether or not there is miscalibration, heterogeneity in linear responses, heterogeneity in quadratic responses, and heterogeneity in taxes (or the relevant stimulus variable). It suggests that heterogeneity in the tax change likely generates a divergence between the two types of models (rows (1)–(3) in Table 1), though endogenous differences (4) and miscalibration (5) and (6) also may play a role.

Representative agent errors are fairly intuitive, and can be broken up into three broad ideas. First, if there are quadratic response terms (assumed here to be negative) variation in taxes will cause a representative agent model to understate the true response by an expression directly related to $\mathrm{Var}(\tau_{\mathrm{i}})$. This is driven by Jensen's inequality giving the mean sum of squares will be higher than the square of the mean. This holds true even if the quadratic response terms are homogeneous. Economically, having two identical people taxed at 50% will have a potentially quite different aggregate response as taxing one individual at 0% and the other at 100%, but this requires nonlinearity in response. Covariance and co-skewness of tax rates with the quadratic term are an additional source of misstatement for a similar reason.

Second, if linear responses are heterogeneous and taxes are heterogeneous, then the representative agent model will under- or overstate the aggregate response depending on the sign and magnitude of $Cov(\beta_i, \tau_i)$, the covariance between responses and tax rates. This too is intuitive: if a government taxes more responsive people more and less responsive people less, the representative agent will understate aggregate responsiveness by shifting taxes to the less responsive and away from the more responsive. While this appears obvious, it may enter a model subtly. For instance, with standard preferences households with more nonlabor income are, ceteris paribus, more elastic in their labor supply. Consequently, models with labor taxes that covary with nonlabor income will display heterogeneous wage elasticities even if households have homogeneous preferences over labor supply. The opposite holds true if a government taxes the less responsive more.

Finally, endogenous response differences and miscalibration of responsiveness operate through a similar channel. The first is a miscalibration of response to a shock, while the other is a miscalibration of the shock itself. As I discuss in the next section, miscalibration of responsiveness due to naive calibration to macroeconomic levels can be significant and is predictable from a representative agent model.

3. Diagnosing divergence between RA models and HA models

Table 1 suggests a way of predicting when models will diverge. Typically, HA models introduce new state variables that are heterogeneous across agents. If responsiveness is a function of these new state variables, β_i and γ_i are likely to be heterogeneous. If the shocks or policies a researcher analyzes covary with state variables that control heterogeneity, even if it is linear, divergences may occur, with the quantitative severity governed straightforwardly in Table 1. For instance, most Bewley-Huggett-Aiyagari-style models with uninsurable idiosyncratic income risk propose individual wealth as an idiosyncratic state variable. However, if labor's responsiveness to wage or tax rates varies with the proposed idiosyncratic state variable of wealth, an issue may arise. If any change in responsiveness occurs that is not uniformly distributed by the state variable, then the second term in Table 1 governs the misstatement. In the example I discuss, because the TCJA had differential effects by proposed idiosyncratic state variables of human capital, wealth and pre-reform tax rates, such a correlation can be of first-order importance.

Second, if the relationship between responsiveness and the exogenous outcome of interest is nonlinear, divergence may occur and is scaled by the variance in responsiveness. For a concrete example, while Krusell and Smith (1998) finds in favor of approximate aggregation, it does not do so because agents in their model have a linear savings rate as a function of income along all incomes. Instead, they find strong nonlinearities in the savings rate for those with low income. The reason approximate aggregation is obtained in spite of nonlinearity in the response function is because the variances of β_i and γ_i are so small as to be near zero–most agents are on the linear portion of responsiveness. Table 1 helps make clear the quantitative role of the low variance in realized β and γ in Krusell and Smith (1998).

From these two observations about Table 1 comes a method of predicting when RA and HA models may diverge using only the RA model. Specifically, a researcher should compute how RA response functions change as a function of variables that would be heterogeneous in an HA model. First, calculate the RA β and γ of interest, either in closed-form or numerically, which is computationally inexpensive compared to a full HA model. Then, examine how β and γ change as a function of proposed HA state variables by perturbing the RA model away from its steady state, as a heterogeneous household might be. For instance, if a HA model of labor supply adopted heterogeneous idiosyncratic wages, checking if $\frac{\partial\beta}{\partial w} = \frac{\partial\gamma}{\partial w} = 0$ would be sufficient to suggest to a second-order approximation that the RA should do a good job for this variable. If the proposed state variable is transitory or perstistent but not permanent, such as a shock to wages, this information can be gleaned by perturbing an RA model with an perfectly transitory out-of-equilibrium wage shock, as both extract information about heterogeneity in the Frisch elasticity. If the proposed state variable is permanent, such as preference differences, re-solving the RA model with the new value and examining the difference in β and γ would be the appropriate comparison. The relevant perturbing variables would include in most modern macroeconomic models idiosyncratic wage, wealth/capital (as in Chang and Kim (2007) but could include discount rates as well, as in Krusell and Smith (1998).

3.1. Analyzing preferences to understand when divergence will occur

In what follows, I offer an example involving common macroeconomic preferences that will predict a divergence between the HA and RA model I implement later. Specifically, I show that miscalibration in common macroeconomic preferences can occur when preferences or prices are chosen to match levels, but affect elasticities. This is prominently the case in the commonly-used "balanced-growth" labor preferences of King et al. (1988) when used for heterogeneous agents off their balanced growth paths. For instance, take constant Frisch elasticity of labor supply preferences with utility U over consumption and labor:

$$U(c_{i,t}, L_{i,t}) = \sum_{t=0}^{\infty} \beta^t \left(\log(c_{i,t}) - \psi \frac{\epsilon}{1+\epsilon} L_{i,t}^{\frac{1+\epsilon}{\epsilon}} \right)$$

Subject to the budget constraint:

$$c_{i,t} + K_{i,t+1} = w_{i,t} L_{i,t}(1-\tau_{i,t}) + r_t K_{i,t}$$

where $c_{i,t}$ is consumption, L_t is labor hours supplied, $K_{i,t}$ is capital holdings, $w_{i,t}$ is idiosyncratic wage, $\tau_{i,t}$ is flat labor income tax rate, r_t is the interest rate, ψ is disutility of labor supply, β is the discount factor, and ϵ is the Frisch elasticity of labor supply. King et al. (1988) note that the income and substitution effects of technological progress cancel along the balanced growth path of the representative agent. This is dependent on nonlabor income rising at the same rate as wage income–if nonlabor income fell proportionally to wage, the substitution effect would dominate, and as wages rose laborers would work more. If nonlabor income rose, the income effect would dominate, and labor would fall as wages rose. While this is ruled out by construction in a typical representative agent model, it is rarely ruled out in a heterogeneous agent model. Typically, nonlabor income does not remain proportional to wage, as agents in a stochastic model are persistently perturbed far away from their balance growth paths, either due to idiosyncratic wage shocks, unemployment shocks, or birth/death shocks. In the language of this paper, β and γ are functions of wealth, even though β and γ are zero at mean (BGP) wealth.

For a tractable closed form example when preferences, rather than prices, cause miscalibration, take the representative agent of Prescott (2004) and a special case of King et al. (1988) preferences. In a static version of the model, households have log utility over both consumption c_i and leisure $\overline{L} - L_i$, where L_i is hours per week worked and $\overline{L}$ is hours per week of free time.

$$u(c_i, L_i) = \log(c_t) + \psi_i \log(\overline{L} - L_i)$$

Household decisions are subject to the household's period budget constraint with individual consumption, labor, and nonlabor income taxes τ_i^c, τ_i^L, and τ_i^v, wage w_i and (fixed) nonlabor income v_i:

$$(1+\tau_i^c)c_i = (1-\tau_i^L) w_i L_i + (1-\tau_i^v) v_i$$

Note that the preferences and budget constraint given above can be preferences for both a representative agent model and individual preferences and budget constraint for a heterogeneous agent model. With the representative agent model in mind, I use this formula to examine in closed form the consequences of a labor income tax reform. Taking first order conditions, labor is given by:

$$L_i^* = \frac{\overline{L} w_i(1-\tau_i^L) - (1-\tau_i^v) v_i \psi_i}{w_i(1-\tau_i^L)(1+\psi_i)}$$

In King et al. (1988), v_i grows proportionately with w_i, as growth rates γ_w and γ_v are the same by assumption, so changes in w would cancel (v is proportional to w). But in a HA model, individuals may diverge from this. To diagnose where an RA model of labor tax reform $\Delta\tau^L$ would diverge from a HA model without first calculating the HA model, one needs to know (for instance) how $\frac{\partial L_i}{\partial \tau_i}$ varies with the potential state variables in a heterogeneous agent model. One particularly pertinent example in the literature is uninsurable labor income risk: a HA model might have heterogeneous idiosyncratic wages, which could be modeled through w_i, and individual capital holdings, which could be modeled through v_i in an incomplete-markets model. To examine the effects of v_i and w_i, β_i and γ_i are required. Given L_i^*, solve for $\beta_i = \frac{\partial L_i^*}{\partial \tau_i^L}$ and $\gamma_i = \frac{1}{2} \frac{\partial^2 L_i^*}{\partial (\tau_i^L)^2}$ in closed form, though numerical exercises around the RA calibration offer the same insight to practitioners:

$$\beta_i = -\frac{v_i(1-\tau_i^v)\psi_i}{w_i(1-\tau_i^L)(1+\psi_i)} \qquad \gamma_i = -\frac{4 v_i(1-\tau_i^v)\psi_i}{w_i(1-\tau_i^L)^3(1+\psi_i)} \tag{8}$$

Equation (8) makes clear that the Marshallian elasticity of labor supply with respect to a change in a household's average tax rates varies considerably, depending on the level of work (determined by $\psi_i/(1+\psi_i)$), and the ratio of property income to post-tax wage $v_i(1-\tau_i^v)/(1-\tau_i^L)w_i$.[4] Equation (8) allows us to clearly see how and why a HA model will diverge. To do so, take the derivative of the response function with respect to the proposed state variables in the heterogeneous agent model, in this case $\frac{\partial \beta_i}{\partial w_i}$:

$$\frac{\partial \beta_i}{\partial w_i} = \frac{v_i \psi_i}{w_i^2(1+\psi_i)} \tag{9}$$

Because β_i in Equation (9) changes nonlinearly as a function of w_i, a mean-preserving spread in w_i will, ceteris paribus, increase β_i by Jensen's inequality if β_i was positive initially.[5] This will lead to a first-order miscalibration when calibrating v and w to their macroeconomic levels, and ψ to the level of labor supply. This alone leads to significant miscalibration when taken to the HA model. Perhaps most importantly, however, Table 1 helps a researcher quantitatively assess whether or not the deviations will be important in practice, rather than via guesswork.

The values for v_i, τ_i^v, τ_i^L, and w_i are measurable, and there is a single free parameter to calibrate, ψ_i. Importantly, there are three obvious and mutually exclusive choices of how to match the representative agent's ψ_R: it can match either:

- The mean labor supply (the usual choice): $\frac{1}{N}\sum_{i=1}^{N} L_i$
- The mean linear responsiveness to a tax change: $\frac{1}{N}\sum_{i=1}^{N} \beta_i$
- The mean quadratic responsiveness to a tax change: $\frac{1}{N}\sum_{i=1}^{N} \gamma_i$

Because of our simple choice of representative agent preferences, a calibration can only match one. Because macroeconomic models frequently calibrate to the levels of variables, such as in this case choosing ψ_i to match the aggregate level of labor $\overline{L}$, it is possible to miscalibrate both $\overline{\beta}$ and $\overline{\gamma}$. Indeed, because wages map to β nonlinearly, this is guaranteed for a representative agent by Jensen's inequality. In such a case, the additional miscalibration errors described in Equation (6) and predicted from the RA model above are potentially important. As I discuss, this miscalibration is potentially exacerbated by discrete choices, such as labor supply. When the level of covariates like wage or hours worked influences elasticities nonlinearly, calibrating to match levels leads to potentially significant deviations between RA and HA models.

[4] In many representative agent macro models, the ratio of wage and property income is constant, causing the Marshallian elasticity to be zero as noted in King et al. (1988). However, in most stochastic heterogeneous agent models, this ratio does not hold for the typical agent, as stochastic shocks keep them from the balanced growth path. Similarly, savings in lifecycle models cause this ratio to not be constant, yielding heterogeneous elasticities.

[5] This is also true when ψ_i is changed so as to hold labor constant.

Table 2
Summary of representative agent misstatement of aggregate response.

Category	Functional Forms	Parameter values
Demographics	$\mu_{j+1} = \mu_j/(1+n)$	$n = 0.01, J = 63, Retire = 43$ $j = 1 \ldots 63$(ages23 − 85)
Technology	$F(K,L) = AK^{\alpha}L^{1-\alpha}$	$(A,\gamma,\delta) = (0.877, 0.352, 0.044)$
Tax system	$T(e;\tau) + \tau_c C + \tau_k kr$ for j < Retire $\tau_c c + \tau_k kr - transfer$ for j ≥ Retire transfer = 18115	$T(e)$ is based on Fig. 2 $\tau_c = 0.10$ and $\tau_k = 0.20$
Preferences	$u(c,n) = \log(c) - \varphi\frac{(n+s)^{1-1/\nu}}{1-1/\nu}$	$\beta = 0.967, \varphi = 0.618, \nu = 0.35$
Human capital	$H(h,s,a) = h(1-\delta_h) + a(hs)^{\alpha}$	$(\alpha,\delta_h) = (0.677, 0.0043)$
Initial	$a \sim PLN(\mu_a,\sigma_a^2,\lambda_a)$ and $\epsilon \sim LN(0,\sigma^2)$	$(\mu_a,\sigma_1,\lambda_a) = (-0.977, 1.67, 3.45)$
Conditions	$\log(h_1) = \beta_0 + \beta_1\log(a) + \log(\epsilon)$	$(\beta_0,\beta_1,\sigma) = (4.68, 0.939, 0.711)$

Table 2: This table replicates the calibration of Badel and Huggett (2017), used in this paper. PLN stands for pareto-lognormal, while LN stands for lognormal.

4. Application

4.1. Model

Having shown in the previous section that a representative-agent model is likely to misstate labor's responsiveness to a tax change In this section, I now take that insight to a fully-fledged model. Specifically, I apply my method to the modern dynamic macroeconomic general-equilibrium model of Badel and Huggett (2017). I adapt their model and estimation method to estimate the difference between a properly-calibrated representative agent model would predict and their heterogeneous agent model's results. For realistic variation in the “treatment” of interest, I estimate the change in tax rates by income for the Tax Cuts and Jobs Act of 2017 and apply them to the rest of the model calibrated by Badel and Huggett (2017).

Badel and Huggett (2017) consider a general equilibrium model of an otherwise standard Ben-Porath (1967) model. In what follows, I give a brief overview of their model. Interested readers can find more details in Badel and Huggett (2017). The authors' model takes the standard overlapping generations model with choices over labor and leisure and adds four new ingredients. First, they introduce human capital. Second, they model idiosyncratic heterogeneity in the ability to accumulate human capital. Third, they introduce the choice to spend time to accumulate human captial, rather than exogenously determining time spent. Finally, they model the labor income tax as a nonlinear piecewise function estimated from NBER's TAXSIM. The authors take their model and calibrate parameters, including the tax code, to the U.S. economy in 2010, though I deviate by updating their tax parameters before and after the TCJA.

Agents (indexed by i) in an overlapping generations model maximize period t utility $u(c_{i,t}, n_{i,t}, s_{i,t})$ from consumption $c_{i,t}$, labor $n_{i,t}$ and time spent on human capital accumulation $s_{i,t}$. They differ in their ability a_i to accumulate human capital $h_{i,t}$, as well as their initial human capital $h_{i,0}$. They maximize utility over the course of their life in the face of progressive and nonlinear taxation as a function of labor earnings $e_{i,t}$, $T(e_{i,t})$. Agents ability to maximize utility is subject to a Ben-Porath human capital production technology $H(h_{i,t}, s_{i,t}, a_i)$ which is a function of human capital and ability as well as time spent accumulating human capital $s_{i,t}$.

The period utility function, discounted at a rate β is given by:

$$u(c_{i,t}, n_{i,t}) = \log(c_{i,t}) - \varphi\frac{(n_{i,t}+s_{i,t})^{1-1/\nu}}{1-1/\nu}$$

Where ν is the Frisch elasticity of substitution and φ is the disutility of non-leisure hours. And is maximized subject to the household budget constraint:

$$c_{i,t} + k_{i,t+1} = w_t h_{i,t} n_{i,t} + k_{i,t}(1+r_t) - T(w_t h_{i,t} n_{i,t}) - \tau_k k_{i,t} r_t - \tau_c C + tf$$

Where $k_{i,t}$ is individual wealth holdings, w_t is the economywide wage rate, $T(\cdot)$ is the labor income tax function, r is the interest rate, and τ_k and τ_c are capital and consumption tax rates, respectively. Households that are retired receive a lump-sum transfer *tf*. The Ben-Porath human capital production technology governs how time spent accumulating human capital maps into human capital next period:

$$h_{i,t+1} = h_{i,t}(1-\delta_h) + a_i(h_{i,t}s_{i,t})^{\alpha}$$

Households live for 63 periods, from age 23–85 and are forced to retire at age 66. Population grows at a 1% rate, and the aggregate production technology is Cobb-Douglas. Table 2, reproduced from Badel and Huggett (2017), denotes parameter choices.

Economy-wide parameters are set to be comparable for the representative agent. I choose representative ability, disutility of labor, and Frisch elasticity to match mean earnings, as well as the linear and quadratic responsiveness of labor to the tax changes. Because I target changes rather than levels, the level of labor is overstated by nearly 20%–this zeroes the effect of miscalibration, but at the cost of misstated labor.

4.2. TCJA 2017 calibration

The authors calibrate their labor income tax rates by calibrating to NBER's TAXSIM calculator, and estimate a piecewise function for marginal tax rates for the year 2010. I replicate their exercise, but update the tax parameters using data on married couples in 2017 and 2019, before and after the TCJA was implemented. Comparable to Fig. 1 in Badel and Huggett (2017), I depict the calibrated effect of the TCJA 2017 on marginal tax rates in my model in Fig. 2. Importantly for this exercise, there is heterogeneity in the estimated change in effective tax rate by income bracket and by previous tax rate. Consequently, the covariance of tax change and responsiveness to tax change is nonzero, yielding a deviation from the representative agent model.

4.3. Results

In this section, I compare the difference between the long-run effects of the TCJA predicted by a Badel and Huggett (2017)-style model and a representative agent. As discussed earlier, the major likely reason for divergence is predicted from a representative agent model, because the reduction in marginal income tax rates from the TCJA is heterogenous by an individual state variable that controls responsiveness (human capital and age). To calculate within-individual long-run difference, I simulate the long-run steady state of both economies for the same population of agents undergoing the same shocks under different tax regimes, so that every compared agent has the same exogenous age/death profile, ability level, and initial human capital, but lives under different tax

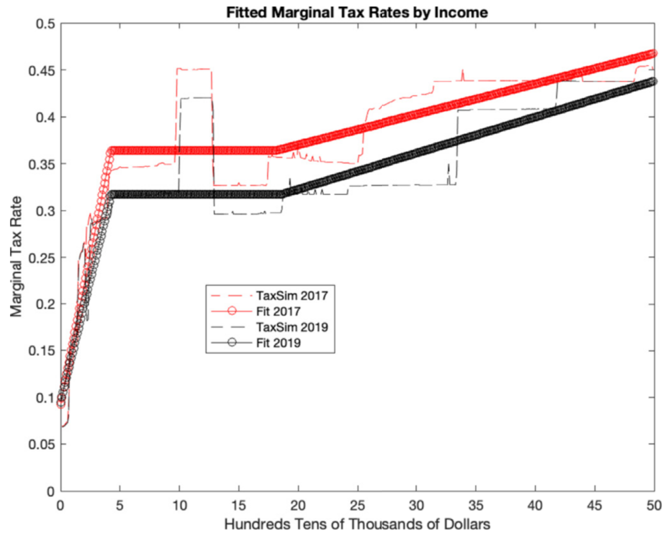

Fig. 2. This figure depicts NBER TAXSIM tax rates and the piecewise fitted marginal tax schedules used for $T(e_{i,t})$. Importantly, the estimated change in marginal tax rates is differential by income. Because income in the Badel and Huggett (2017) lifecycle model is correlated with labor supply elasticity, a representative agent calibrated to the mean will understate the true effect.

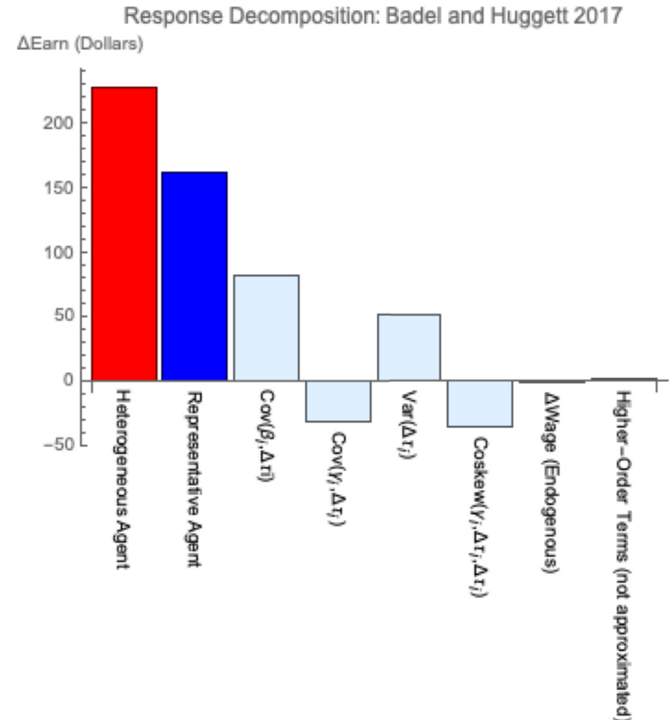

Fig. 3. This figure decomposes the differences between a properly-calibrated representative agent model and the heterogeneous agent model of Badel and Huggett (2017). The light blue lines are the decomposition terms described in Table 1, and their sum adds to the difference between the solid red and solid blue lines.

regimes.[6] I then compare the effect of the tax cut on average earnings for each model, and decompose the difference.

Because I calculate the differences between long-run steady states, there is some change in labor supply due to a change in wage rates, not just marginal tax rates. Fortunately, the baseline effect on wages is negligible: a 0.2% change. I include endogenous effects from changes in the wage via Equation (7) with the rest of the standard difference described in Equation (5) for completeness, though differences between the models due to changes in wage rates is insignificant in this case.[7]

Fig. 3 depicts the results of decomposing the difference between a heterogeneous and representative agent simulation of the long-run effect of the TCJA on household earnings. The representative agent model predicts a $161 increase in taxable earnings due to the tax cuts. However, the heterogeneous agent model predicts a $228 increase in taxable earnings, a 41% increase above the representative agent's predictions. As predicted from our analysis of representative agent preferences, the lion's share of the divergence comes from the covariance between effective labor supply responsiveness and tax rate change: while the difference between the two is $67, the covariance between taxes and linear responsiveness would have predicted a difference of $82. The importance of not simply linearizing the difference becomes clear from the terms involving γ, the quadratic term. Collectively these reduce the difference by $15, or 20% of the initial difference. As mentioned, the very small long run endogenous change in wages produces a change in earnings of less than a dollar ($0.90).

The central contribution of this paper is to show that Table 1 may be used to decompose differences between RA and HA models. This decomposition is given in Fig. 3, and can provide a useful guide to both where to look for causes of divergence and a method to decompose differences when they occur. As we have seen in the example of a Badel and Huggett (2017)-style model analyzing tax changes, each of the main decomposition terms has the capacity to be a significant contributor of divergence. In this example, the cause of largest divergence between HA and RA was diagnosable from the analysis of preferences that this paper suggests: take a standard agent and analyze whether or not their responsiveness changes as a function of parameters that would vary in a heterogeneous-agent model (such as tax rates and wealth).

5. Conclusion

Representative agent models are surely convenient, and allow a clear understanding of the facets of data that matter. However, this clarity comes at a cost: when agent responses are nonlinear or heterogeneous in variables changed by policy, and when the affects of policy are heterogeneous, the representative agent model may fail to properly capture the full agent's response. This paper offers a simple formula to understand when such divergences might occur, to diagnose whether they will occur in a given RA model, and to categorize their sources when they do occur. To shed light on how to use this decomposition, I take a modern dynamic macroeconomic model of human capital accumulation based on Badel and Huggett (2017), examine its preferences and budget constraint, and predict how it would diverge from a representative agent model. I find, consistent with the predictions of a representative agent model, a significant divergence due to the correlation of linear responsiveness and tax rates.

More broadly, this paper emphasizes that the practice of taking "representative" budget constraints is only valid when the derivative of responsiveness with respect to heterogeneous state variables is zero, or when it is equal to a constant and a multiplicative free parameter is chosen to re-align RA responsiveness with mean HA responsiveness. Moreover, it allows researchers to examine RA models for the potential causes of divergence by first perturbing the model with respect to proposed heterogeneous variables and quantitatively understand the potential severity of these deviations. Finally, it shows how to linearly

[6] I also experimented with deficit-financed tax cuts via a single-period reduction in tax rates followed by a small increase in tax rates thereafter, and found comparable results. For clarity and to avoid repetition, I do not show these results.

[7] I also ran a model in which imposed a one-period tax change that was reversed in the subsequent year, reflecting a financial retrenchment in which government slightly raised taxes in all future periods to pay for the net present value of the 2017 tax cut. The figure was qualitatively similar to 3, and so is omitted.

decompose the sources of model divergence when they do occur.

While this paper's approach has advantages, it also possesses limited applicability in more complicated macroeconomic models that involve interactions between state variables. While a quadratic approximation may do a good job summarizing an agent's response to a shock in the first few periods of a dynamic model, approximation errors due to higher-order effects may compound over time, and may be unable to capture the effects of multiple interacting state variables on responsiveness, reducing the amount of difference between HA and RA models explained by my formulae. As a consequence, a researcher may be well-served in using economic reasoning as a first step in model selection, and after determining model suitability may use the formulae outlined in this paper.

In the case of standard labor preferences, labor responsiveness to tax changes is highly nonlinear with respect to relevant heterogeneous state variables, such as assets or idiosyncratic labor. Diagnosing this further, one strength of this paper's approach is the ability to predict the sources of divergence and decompose them when they arise. In the case of a dynamic model of labor supply response to tax changes, we saw that introducing heterogeneous wealth into standard preferences strongly affected responsiveness of an HA model relative to an RA model. Even accounting for miscalibration, the source of heterogeneity in responsiveness (wealth relative to wages and age) combined with the tax policy, which was not uniform across income ranges, caused a strong divergence. While no tool is perfect, the method embodied in Equations (4)–(7) joins a suite of varying approaches, such as Boppart et al. (2018) and Assenza and Gatti (2013) in advancing our understanding and modeling the difference between heterogeneous agent and representative agent of heterogeneous agent models.

Declaration of competing interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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